FINAL REPORT ON GRANT N0014-90-J-1324 SUBMITTED TO THE OFFICE OF NAVAL RESEARCH Arlington, Virginia

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Solving large-scale combinatorial optimization problems

Short summary of research under Grant N0014-90-J-1324

Optimization problems are concerned with the efficient use or allocation of limited resources to meet desired objectives. These problems are characterized by the large number of alternatives that satisfy the basic conditions of each problem. The selection of a particular solution as the best solution to a problem depends on some goal or over-all objective. The versatility of the combinatorial model stems from the fact that in many practical problems, activities or resources, such as machines, airplanes, missile target sites, and people are indivisible. Also, many problems have only a finite number of alternative choices and consequently can appropriately be formulated as combinatorial problems. We refer the reader to the following texts and their bibliographical references for further review of some of these important engineering and managerial decision problems: *Combinatorial and Integer Programming* (Nemhauser and Wolsey), *Applied Mathematical Programming* (Bradley, Hax and Magnanti), *Principles of Operations Research* (Wagner), and *Model Building in Mathematical Programming* (Williams).

The general class of combinatorial optimization problems we have studied and are relevant to the strike-force planning problem is of the the form of a pure (0,1) linear programming problem, which we shall refer to as *Problem* Z_{IP}:

$$\begin{array}{ll} \min\limits_{x} & cx \\ \text{subject to} & Ax \leq b; x \in \{0,1\} \end{array}$$

where A is an mxn matrix, b is an m-vector and c is an n-vector.

In its general form, *Problem* Z_{IP} belongs to the class of NP-complete or hard problems for which to date no technically good (i.e. polynomially bounded) methods of solution are known. While in terms of worst-case analysis, *Problem* Z_{IP} may remain insoluble, the empirical record of optimizing specific instances of large-scale problems is encouraging: see Crowder, Johnson and Padberg [1983], Grötschel, Junger and Reinelt [1984], Barahona and Maccioni [1982], Martin and Schrage [1985], Martin [1984], Van Roy and Wolsey [1985], Balas and Ng[1986], and Leung and Magnanti [1986], Dietrich and Escudero [1988], Monma and Grötschel[1990], and Padberg and Rinaldi[1990] and Hoffman and Padberg[1992] for specific problem classes successfully solved and Hoffman and Padberg [1985] and Padberg and Rinaldi[1990] for a survey and overview of such approaches. Each of these papers uses cutting planes based on the polyhedral structure of integer polytopes to tighten the linear programming relaxation of *Problem* Z_{IP}. When it is not possible to generate any further cuts (due to an incomplete understanding of the polyhedral structure, or due to an inability to generate cuts of a known form *algorithmically*), the algorithms resort to *branch-and-cut* – the use of cutting planes based on polyhedral theory into a general search-tree framework whereby cutting planes are not only used prior to branching but also within the search tree. For details on the development of branch-and cut, see the papers by Padberg and Rinaldi [1987,1991], and Hoffman and Padberg [1989,1992, 1993,1995] and extended to a parallel algorithm in Cannon and Hoffman [1989].

It had been presumed until recently that problems having more than a few hundred decision variables could not be solved to optimality, and so heuristics were usually employed producing, in general, *suboptimal* solutions, i.e. feasible solutions without any guarantee as to their "closeness" to a best possible problem solution. Results of research performed under this grant have shown that problems having thousands, and sometimes millions, of variables can be solved using present-day technology based on mathematical results that utilize the structure underlying the problem and that incorporate related advances of the mathematical theory into the general approach called branch-and-cut. Such algorithms are now available in commiercial software such as OSL [IBM, 1992] and Cplex [Cplex, 1993].

Our computational successes to date have enabled us to acquire a variety of real-world problems. when it was believed that one could only solve integer programming problems having a few hundred variables, industry and government organizations proceed to use "ad hoc" solution procedures and avoided the careful modeling of the "true" optimization problem. Thus data was scarse and it was difficult to know which problem areas were important to the applied community. It is precisely the successes of the past few years that allows us entry into the applied defense community. Seeing new problem areas provides us with a stimulus for further mathematical developments which, in turn, allows the solution of a larger set of important combinatorial optimization problems.

Publications under ONR Grant N00014-90-J-1324

Books published:

Padberg, M. (1995) *Linear Optimization and Extensions* Springer-Verlag, Berlin. 448 pp.

Papers published in refereed journals:

Alevras, D., M. Padberg and M.P. Rijal "The convex hull of a linear congruence relation in zero-one variables" (1995) *ZOR-Mathematical Models of Operations Research* **41** 1-23.

Alevras, D. and M. Padberg (1994) "Order preserving assignments" (1994) *Naval Research Quarterly*

Cannon, T.L. and Hoffman, K.L. (1990) "Large-scale linear programming on distributed workstations", *Annals of Operations Research* **22** 181-217.

Domich,P.D., Hoffman, K.L. Jackson, R.H.F. and McLane, M. (1991) "A microcomputer based solver for the facility location problem" *Management Science.* **37** 960-979.

Harris,C, Hoffman K.L. and Yarrow, L.A. (1995) "Obtaining ptimal latin hypercube sampling plans" *Annals of O.R.* **58** 243-260.

Harris, C. Hoffman, K.L. and Yarrow, L.A. (1995) "Obtaining minimum-correlation latin hypercube sampling plans using an ip-based heuristic" *OR Spektrum* **17** 139-148

Hoffman, K.L. and Padberg, M. (1991) "Techniques for improving the LP-representation of zero-one linear programming problems" *ORSA Journal on Computing*, **3** 121-134.

Hoffman, K.L. and Padberg, M. (1994) "Solving airline crew scheduling problems by branch-and-cut" *Management Science.* **39** 657-682.

Hoffman, K.L. and Padberg, M. (1995) "Combinatorial and Integer Programming" *Encyclopedia of Operations Research*, Springer-Verlag.

Hoffman, K.L. and Padberg M. (1995) "The Traveling Salesman Problem" *Encyclopedia of Operations Research*, Springer-Verglag.

Hoffman, K.L. and Padberg M. (1995) "Bestimmung optimaler Einsatzplane fur Flugpersonal" *Mathematic in der Praxis* Springer-Verlag. 509-532.

Padberg, M. (1994) "Lehman's forbidden minor characterization of ideal 0-1 matrices" *Annals of Discrete Mathematics 111* 409-420.

Padberg, M. and Rinaldi, G. (1989) "A Branch-and-Cut Approach to a Traveling Salesman Problem with Side Constraints" *Management Science* **35**,1393-1412

Padberg, M. and Rinaldi, G. (1990) "An efficient algorithm for the minimum capacity cut problem" *Mathematical Programming Series A*, **47**, 19-36.

Padberg, M. and Rinaldi,G (1991) "A branch-and-cut algorithm for the resolution of large-scale symmetric traveling saleman problems" *SIAM Review* **33**, 60-100.

Padberg, M. and Rinaldi, G. (1990) "Facet identification for the symmetric traveling saleman polytope" *Mathematical Programming* **47** 219-257.

Padberg, M. (1989) "The Boolean Quadric Polytope: Some Characteristics, Facets and Relatives" *Mathematical Programming, Series B*,**45** 139-172.

Padberg, M. and Sung, T-Y. (1991) "An analytical comparison of traveling salesman problems", *Mathematical Programming.* **52** 315-358.

Padberg, M. and Sung, T-Y. (1993) "An analytical derivation of max-flow min-cut" *Mathematical and Computer Modelling*

Padberg, M. and M. Wilczak (1993) "Boolean polynomials and set functions", *Mathematical and Computer Modelling* **17** 3-6.

Papers submitted to refereed journals (and not yet published):

Alevras, D. and Padberg, M. "Small min-cut polyhedra" submitted to *Mathematics of Operations Research*

Rushmeier, R., Hoffman, K. and Padberg, M. "Solving large scheduling problems arising in the airline industry" submitted to *Operations Research*

Alevras, D. and Padberg, M. "Order preserving assignments without contiguity" submitted to *Discrete Mathematics*

Honors/Awards/Elected Offices:

Dr. Karla L. Hoffman was awarded the Distinguished Faculty Award by the President and Board of Visitors of George Mason University, on April 30, 1989. Each school within the university honors one faculty member per year with this award. She has served as Treasurer of ORSA, 1993-1994 and now serves as Treasurer of INFORMS, 1995-1996. She is Associate Editor of *Mathematical Programming, Series B, SIAM, Journal on Optimization, Computational Mathematical Programming and Applications.*

Karla Hoffman has been invited to give a tutorial on new developments in Integer Programming solution technologies at the INFORMS Computer Science Technical Section meeting in Dallas TX, January, 1996.

In 1990, Dr. Manfred Padberg received an Alexander-von-Humbolt Research Award for senior U.S. scientists. He was also invited to work at the Ecole Polytechnique during his sabbatical leave from New York University (academic year 1989-1990). During his stay in Paris, Dr. Padberg twice taught a course in combinatorial optimization in the graduate program run jointly by Ecole Polytechnique and the University of Paris (Sorbonne). He also accepted a position as Visiting Distinguished Research Professor at George Mason University, and resides at George Mason University during the summer terms.

Manfred Padberg gave an invited plenary talk on Integer Programming techniques at the ORSA/TIMS meeting in San Fransisco, Fall, 1992. He lectured on combinatorial otpimization at Berlin Technical University and at Bonn Unviversity.